REPORT DOCUMENTATION PAGE Dist: A

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	October 31, 1994	Final Report – 1 Oct. 1990–30 Sept.1994

4. TITLE AND SUBTITLE

COMPUTATIONAL FLUID DYNAMICS and TRANSONIC FLOW

5. FUNDING NUMBERS

Grant AFOSR-91-0042

6. AUTHOR(S)

Paul R. Garabedian

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

New York University
Courant Institute of Mathematical Sciences
251 Mercer Street
New York, N.Y. 10012

8. PERFORMING ORGANIZATION REPORT NUMBER

AFOSR-TR· 94 0760

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)

AFOSR/NM, Building 410
110 Duncan Avenue, Suite B115
Bolling AFB DC 20332-0001

Dr. Marc Q. Jacobs/NM (202) 767-5025

10. SPONSORING / MONITORING AGENCY REPORT NUMBER

DTIC SELECTED DEC 2 7 1994 F

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION / AVAILABILITY STATEMENT

12b. DISTRIBUTION CODE

A

13. ABSTRACT (Maximum 200 words)

The project was concerned with the development of high performance computer codes for problems in transonic aerodynamics. A practical rule to calculate the wave drag for solutions of the Euler equations was developed from an entropy equality. Symmetric shockless airfoils were analyzed for which uniqueness fails in the transonic case not just for potential flow, but also for the Euler equations.

19941221 035

DTIC QUALITY INSPECTED 1

14. SUBJECT TERMS

High performance computing; transonic aerodynamics; wave drag; uniqueness.

15. NUMBER OF PAGES

4

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	UL

Grant AFOSR-91-0042

COMPUTATIONAL FLUID DYNAMICS and TRANSONIC FLOW

FINAL TECHNICAL REPORT

Period: 1 October 1990 - 30 September 1994

By

Paul R. Garabedian

New York University
Courant Institute of Mathematical Sciences
251 Mercer Street
New York, N.Y. 10012

The addition of artificial viscosity terms to the Euler equations not only ensures numerical convergence, but also models the physical problem correctly. Specifically, the additional terms guarantee that an entropy inequality is satisfied. This can be achieved by upwind differencing in the supersonic region of a transonic flow so that the truncation error represents an artificial viscosity satisfying the entropy inequality. Most Euler solvers, however, employ central differences for all spatial derivatives, and therefore explicit artificial viscosity terms are added to all four equations in order to guarantee convergence. A check on the physical validity of the additional terms is to show that not only is the entropy inequality satisfied for the modified equations, but it also results in a formula for the wave drag depending on the specific form of the artificial viscosity.

Both potential and Euler solvers in our transonic aerodynamics codes make use of the entropy inequality to measure the wave drag. This calculation involves summing a positive definite quantity over the region of flow in which the shock is smeared, avoiding the use of computed flow variables near the trailing edge where there is uncertainty. In more standard approaches the pressure is integrated around the airfoil or over a contour around the shock, but the double integration is more accurate.

Consider the modified Euler equations

$$\rho_t + (\rho u)_x + (\rho v)_y = \nabla \cdot \nu \nabla \rho$$

$$(\rho u)_t + (\rho u^2 + p)_x + (\rho u v)_y = \nabla \cdot \nu \nabla (\rho u)$$

$$(\rho v)_t + (\rho u v)_x + (\rho v^2 + p)_y = \nabla \cdot \nu \nabla (\rho v)$$

$$(\rho E)_t + (\rho u H)_x + (\rho v H)_y = \nabla \cdot \nu \nabla (\rho H) \, ,$$

where artificial viscosity has been added on the right of the form

$$\nabla \cdot \nu \nabla \equiv \frac{\partial}{\partial x} \left(\nu^{(x)} \frac{\partial}{\partial x} \right) + \frac{\partial}{\partial y} \left(\nu^{(y)} \frac{\partial}{\partial y} \right) \, .$$

1

These equations may be combined to form a fifth equation for the entropy

$$(\rho S)_t + (\rho u S)_x + (\rho v S)_y = \frac{1}{T}\{\nabla \cdot \nu \nabla (\rho H) - (E - TS)\nabla \cdot \nu \nabla \rho$$
$$-u(\nabla \cdot \nu \nabla(\rho u) - u\nabla \cdot \nu \nabla \rho) - v(\nabla \cdot \nu \nabla(\rho v) - v\nabla \cdot \nu \nabla \rho)\} \ ,$$

whose left-hand side is in conservative form. The right-hand side, however, may be separated into a positive definite term and a divergence term leading to wave drag norms of the form

$$(\rho S)_t + (u\rho S)_x + (v\rho S)_y = \|\nu\,\nabla U\|^2 + \|\lambda\,\Delta U\|^2 + \nabla \cdot (\nu S\nabla \rho + \cdots) \ .$$

An application of the divergence theorem then yields a representation

$$C_D = \int\int \left(\|\nu\,\nabla U\|^2 + \|\lambda\,\Delta U\|^2\right) dx\,dy$$

for the wave drag as an integral of positive terms over any region including the shocks. A corollary of this analysis is that the numerical solution of the Euler equations has to satisfy the entropy inequality asserting that S increases through shocks. The entropy integral for the drag has the advantage in three-dimensional flow that it enables one to distinguish the wave drag from the induced drag, which may be much larger. On a practical mesh it turns out that the entropy formula provides a good estimate of drag in the realistic range of Mach numbers.

Methods for the design of supercritical wing sections have led to lifting airfoils for which two distinct solutions of the Euler equations can be found with visibly different shocks at identical flow conditions. Recently we have applied complex characteristics to construct a symmetric shockless airfoil that has lift at zero angle of attack caused by shocks on the upper and lower surfaces that are not the same. Here again the solution of the Euler equations is not unique. The large size of the supersonic zones swept out by the characteristics is presumably what accounts for the existence of multiple solutions at off-design conditions.

Publications

K. McGrattan, Comparison of transonic flow models, AIAA Journal **30** (1992) 2340–2343.

P. Garabedian, Failure of uniqueness in transonic flow, Appl. Numer. Math. **10** (1992) 231–234.

P. Garabedian, A simple proof of a simple version of the Riemann mapping theorem by simple functional analysis, Amer. Math. Monthly **98** (1991) 824–826.

P. Garabedian, Comparison of numerical methods in transonic aerodynamics, Computers Fluids (1993) 323–326.

P. Garabedian, The method of complex characteristics for transonic flow, submitted for publication.

M. Bledsoe and P. Garabedian, Sobre soluciones débiles de la ecuación de Burgers, submitted for publication.

REPORT DOCUMENTATION PAGE

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE October 31, 1994	3. REPORT TYPE AND DATES COVERED Final Report – 1 Oct. 1990–30 Sept.1994

4. TITLE AND SUBTITLE COMPUTATIONAL FLUID DYNAMICS and TRANSONIC FLOW	5. FUNDING NUMBERS Grant AFOSR-91-0042
6. AUTHOR(S) Paul R. Garabedian	

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) New York University Courant Institute of Mathematical Sciences 251 Mercer Street New York, N.Y. 10012	8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) AFOSR/NM, Building 410 110 Duncan Avenue, Suite B115 Bolling AFB DC 20332-0001 Dr. Marc Q. Jacobs/NM (202) 767-5025	10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT	12b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)

The project was concerned with the development of high performance computer codes for problems in transonic aerodynamics. A practical rule to calculate the wave drag for solutions of the Euler equations was developed from an entropy equality. Symmetric shockless airfoils were analyzed for which uniqueness fails in the transonic case not just for potential flow, but also for the Euler equations.

14. SUBJECT TERMS High performance computing; transonic aerodynamics; wave drag; uniqueness.			15. NUMBER OF PAGES 4
			16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT Unclassified	18. SECURITY CLASSIFICATION OF THIS PAGE Unclassified	19. SECURITY CLASSIFICATION OF ABSTRACT Unclassified	20. LIMITATION OF ABSTRACT UL